Team Control Number

12821

Problem Chosen

A

2022

HIMCM

Summary Sheet

Shedding Light on the Population of the Honeybee

It is difficult to envisage a world without honeybees. Honeybees have an irreplaceable role in agricultural production and the maintenance of natural ecosystems. However, honeybee populations are diminishing worldwide. The global catastrophe of honey-bee colony losses is greatly exacerbated by parasites, diseases, inadequate nutrition, pesticides, and climate change. There is a pressing need for understanding the key factors and crucial processes for raising colony survival rates.

The team is tasked with developing a model to determine the population of a honeybee colony over time. To begin with, several assumptions, on which the teams models will be based, are listed and justified to simplify the modeling process. The entire modeling process is separated into three parts: (1) the baseline model is designed as the backbone and contains the Allee effect, which establishes a correlation between population size and the mean individual fitness of the colony population; (2) then we introduce virus into the baseline model and showcase different scenarios under which the honeybee population will either go extinct or stabilize to a steady state; (3) lastly, we introduce seasonality into the baseline model and analyze the effects of time-varying lifespan and number of eggs laid.

We conducted a sensitivity analysis to identify the parameters that have the most impacts on honeybee colony growth and to assess the stability and validity of our model. The parameters selected include natural death rate, lifespan, number of eggs laid by the queen bee, the Allee constant K, equilibrium population, and virus transmission rate and death rate due to virus. By changing these parameters, we examined their impacts on the model outcomes. We discovered that while the average number of eggs the queen lays per day has a substantial impact on our model, the longest lifespan has the greatest impact , where a 10% increase resulted in the average long-term population to increase 30%. This is due to the accumulating population caused by the two factors. Moreover, we have evaluated how viruses can result in extinction and how our seasonality and virus models align with our baseline model.

The relationship between pollination processes and honeybee hives is also deemed important. To predict how many honeybee hives are required to support the pollination of a 20-acre parcel of land containing crops that benefit from pollination, we considered types of crops, temperature, air pollution, and pesticide as factors influencing the process.

An infographics is designed to introduce our website, which offers the information we have compiled. Readers can easily comprehend the most updated honeybee population model in the infographics. They will soon gain insight into our models, which we utilize to offer insightful information on crucial processes and probable factors that could improve colony survival rates, and assist to tackle the dire strait of honeybees.

Key Words: Allee Effect; Honeybees; Dynamic Model; Seasonality; Virus; Pollination

Contents

1 Introduction

1.1 Background

Colony Collapse Disorder (CCD) was coined in 2007 to characterize the global collapse in honeybee colonies [27]. These colonies play a crucial role in agricultural productivity and the maintenance of natural ecosystems [21]. Bee pollination is vital for producing one-third of the food people consume. In the US, the value of honeybee pollination is from $15 to $20 billion each year [10]. Beekeepers have lost between 30 and 45 percent of their colonies each year for the past ten years, with the most recent losses in 2019-2020 topping 40 percent [7]. The beekeepers promptly replace the lost colonies, therefore even though there are progressively more lost colonies, the overall population has not diminished [14]. Moreover, beekeepers may utilize cold storage to assist the colony in successfully surviving the winter [9]. In the context of apiculture, these treatment procedures come with a hefty price. Therefore, the decline of the honeybee colony represents an issue for the world's economy, agriculture, and environment.

Honeybees are social insects that reside in colonies comprised of a single egg-laying queen, zero to several thousand reproductive males, tens of thousands of reproductively sterile female workers, and 10,000-30,000 eggs, larvae, and pupae [3] [15] [26]. Thousands of worker bees work together to construct nests, gather food, and raise young. With respect to their level of adulthood, each member has a specific obligation to fulfill. But the entire colony must work together to survive and reproduce[23]. A worker honeybee's life cycle typically includes the following stages: egg, larva, pupa, capped, adult, and foraging.

We aim to develop a model that can accurately and effectively predict the population of a honeybee colony over time by taking into account a variety of factors, including the population size, the lifespan of a honeybee, virus infection, and climate change with regards to the seasons. The survival rate of colonies can be enhanced by using this model, which sheds light on critical processes and probable contributing elements. This model assists in addressing the economic, environmental, and agricultural crisis brought on by the reduction of the honeybee colonies.

1.2 Restatement of Questions

Our research focuses mostly on the Colony Collapse Disorder (CCD) concern, which causes honeybee populations to drop globally. Hence, building models to estimate honeybee populations is our primary objective.

- A continuous-time dynamic model should first be constructed to describe a honeybee colony's (a honeybee hive's) population over time. The variables that could affect the population of honeybees should be included in the model. For instance, the number of births during a certain time frame may be impacted by egg-laying rates.
- Second, we need to conduct a sensitivity analysis on the population model to see how much each factor affects the size of the honeybee colony. We will alter each factor while holding the others constant and then quantitatively examine how it impacts the colony population. The factors can then be ranked in order of impact.
- Third, we need to create a model to estimate how many honeybee hives are required to pollinate a 20-acre plot of land with crops in the best possible way.
- Last but not least, we would develop a one-page, non-technical blog for a website to present the data we have discovered during our research. This blog may address how a honeybee colony's population will evolve over time and aims to demonstrate our model's findings and their practical application to non-technical readers.

1.3 Assumptions and Justifications

Several assumptions are listed to simplify the problem. The assumptions and their justifications are listed below.

- **Assumption 1:** There is an Allee effect within the honeybee population where the survival probability of the new brood increases in the total population.

  Justification: Colony Collapse Disorder needs to be considered, so the population isn't merely growing but has the possibility to collapse into extinction.

- **Assumption 2:** We consider the honeybee population from a macroscopic view, regardless of each age-stages.

  Justification: As we are interested in the overall population, various age-stages are abstracted from the model.

- **Assumption 3:** We assume that the honeybee population does not have natural enemies and can only die out of natural causes or infections.

  Justification: We consider honeybee colonies under a hospitable environment and ignore the factors of predators and natural disasters for simplification.

2 Population Model

2.1 Variables

Variable Symbol	Meaning
$N(t)$	Population size at time t
N_C	Critical Population
N_{max}	Maximum Sustainable Population
$\sqrt{K}$	Colony size where brood survival rate is half maximum
R	Average number of eggs laid by the queen bee per day
λ	Probability of natural individual death per day
t	Time in days
m	Average lifespan of the honeybee
m_{max}	Longest lifespan of the honeybee in one year period
m_{min}	Shortest lifespan of the honeybee in one year period
t_0	Starting date of population change
φ_0	Time interval between t_0 and the last 22 December
a_m	Amplitude of the lifespan function
R_{max}	The maximum number of eggs laid per day
R_{min}	The minimum number of eggs laid per day
a_R	Amplitude of the fertility function
$N_I(t)$	Population infected with the virus and are themselves infectious
$N_S(t)$	Population that is susceptible to the virus
α	Transmission rate
μ	Probability of recovery when infected
d_I	Probability of death due to infection

Table 1: Variables in the population model

2.2 Baseline Model

2.2.1 Model Setup

Prior to creating a model that incorporates all the factors that influence a honeybee colony's population over time(in days), a baseline model ought to be constructed. This model's assumption applies to all the models developed afterward.

The social organization of honey bees includes the reproductive division of work where each honey bee colony is comprised of a queen (reproductive female responsible for laying eggs), workers (non-reproductive females), and drones (males). Additionally, the number of deaths per unit of time is proportionate to the population. These two factors are the major components of the population change in the honeybee colony.

To begin with, consider the population size $N(t)$ at time t where time is in the unit of days, and we have the population growth per unit time to be:

$$\frac{dN}{dt} = \underbrace{\frac{N^2}{K + N^2}R}_{\text{Birth in unit time (per day)}} - \underbrace{\lambda N}_{\text{Death in unit time (per day)}} \tag{1}$$

The birth of new honeybees is assumed to be the the average number of eggs laid by the queen per day R, multiplied by the survival rate of the brood $\frac{N^2}{K+N^2}$. The survival rate is determined by the population size $N(t)$ at time t, together with the carrying capacity of the population K. Thus the total new birth per unit time can be written as $\frac{N^2}{K+N^2}R$.

λ can be treated as the probability of individual death per day or from the entire population's point of view the proportion or rate of death per day. So the death in unit time may be represented as a constant proportion of the total population, which is λN.

2.2.2 Allee Effect

The Allee effect, a phenomenon in biology marked by a link between population size or density and the mean individual fitness of a population or species [4], is illustrated by this model. When the population N grows to infinity, the survival probability of new brood $\frac{N^2}{K+N^2}$ reaches its approaches to 100% and the number of newborns is consequently approaching the number of eggs laid R. Conversely, as N shrinks to zero, the survival probability of new brood $\frac{N^2}{K+N^2}$ declines to 0, reducing the birthrate. Due to the constant death rate λN, the nonlinear term $\frac{N^2}{K+N^2}$ has an influence on the system that results in an Allee threshold, or critical population, below which the colony will collapse over time and above which it will be able to survive and grow over time.

This applies to the impacts of collaboration in honeybee colonies, where larger colonies result in stronger cooperative efforts between the hive and foraging bees to ensure the survival of young bees. The term $\frac{N^2}{K+N^2}$ represents the probability of an egg turning into an adult. It also denotes that for an egg to develop into a worker bee, care must be provided by adult workers (N) inside the colony, as well as food brought in by foragers. Additionally, this term implies that having more adult workers in the colony can boost the survival of an egg and its development into an adult. Due to the constant death rate λN, the nonlinear term $\frac{N^2}{K+N^2}$ has an influence on the system that results in an Allee threshold, below which the colony will collapse and above which it will be able to survive.

2.2.3 Critical and Maximum Sustainable Population

There are two steady-states in our baseline model. Setting the right-hand side of Equation 1 to be 0, we have

$$\frac{N^2}{K + N^2}R - \lambda N = 0$$

This can be rearranged into a quadratic equation for N.

$$\lambda N^2 - RN + \lambda K = 0$$

The smaller solution to the above quadratic equation yields the critical population of the honeybee colony.

$$N_C = \frac{R - \sqrt{R^2 - 4\lambda^2 K}}{2\lambda} \tag{2}$$

On the other hand, the larger solution yields the maximum sustainable population of the honeybee colony.

$$N_{max} = \frac{R + \sqrt{R^2 - 4\lambda^2 K}}{2\lambda}$$

2.2.4 Parameter Calibration and Model Results

We set λ to be 0.015, indicating an average lifespan of $1/\lambda \approx 67$ days. R is set to be 1500 eggs per day. K is determined by the critical population given by Equation 2 (when $\frac{dN}{dt} = 0$). We take the critical population to be 1000, plug in the values of λ and R into Equation 2, and we get that $K = 9.9x10^7$.[5] This means that when the population of a colony reaches $\sqrt{K} \approx 9950$, the survival probability of new brood becomes 50% [11].

We analyzed the population growth with and without the Allee effect in order to validate the significance of the Allee effect on our model. The average number of eggs laid by the queen bee per day R, death rate λ, and K respectively are 1500, 0.015, and $9.9x10^7$. We used 6 characteristics of the model to conduct analysis: critical population, maximum population, time to reach 90% of the maximum sustainable population, and change in population in 1^{st}, 2^{nd}, and 3^{rd} year.

Figure 1 illustrates the short-term effect of the Allee effect in our model, introduced by the term of the survival probability $\frac{N^2}{K+N^2}$. When the initial population is comparatively small, a lower survival probability results in a lower fertility rate than the ideological value, so the rate of population expansion is lower when the Allee effect is incorporated into the model than it would be without it.

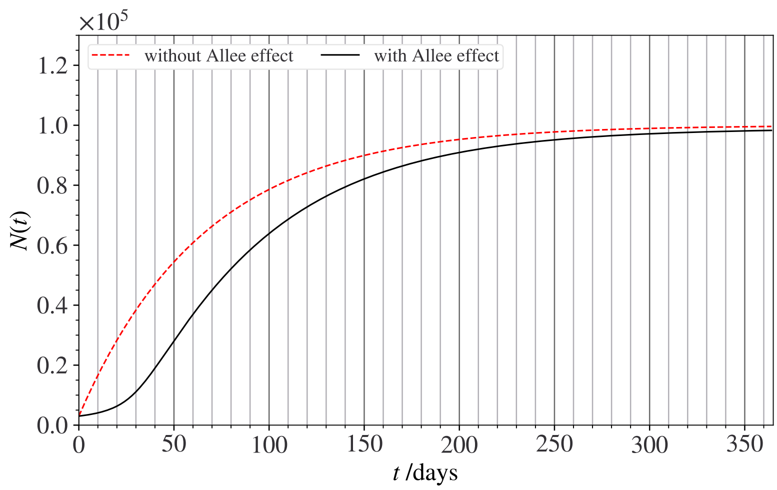


Figure 1: Population change in 1 year starting at 3000 with Allee effect

Figure 1 illustrates population growth for a longer time. The Allee effect has a substantial impact by the first year with a percentage change compared to the initial population of 3176.19%, whereas the two lines are about parallel in the second and third year (3199.89% and 3200.00%, which is the percentage change compared to the initial population at day 1), showing a relatively minor impact. This is due to the carrying capacity having been reached and K's importance decreases as N increases in $\frac{N^2}{K+N^2}$. The time to reach 90% maximum population is 187 days as shown below, indicating the growth of the population.

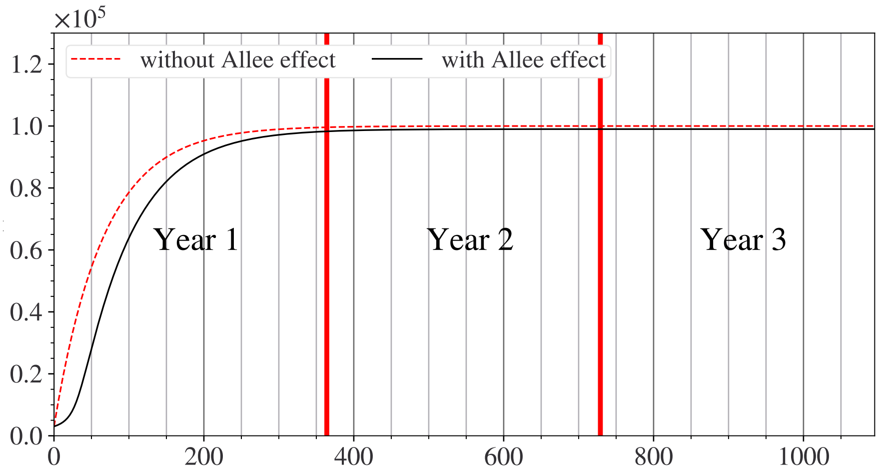


Figure 2: Population change in 3 years starting at 3000 with Allee effect

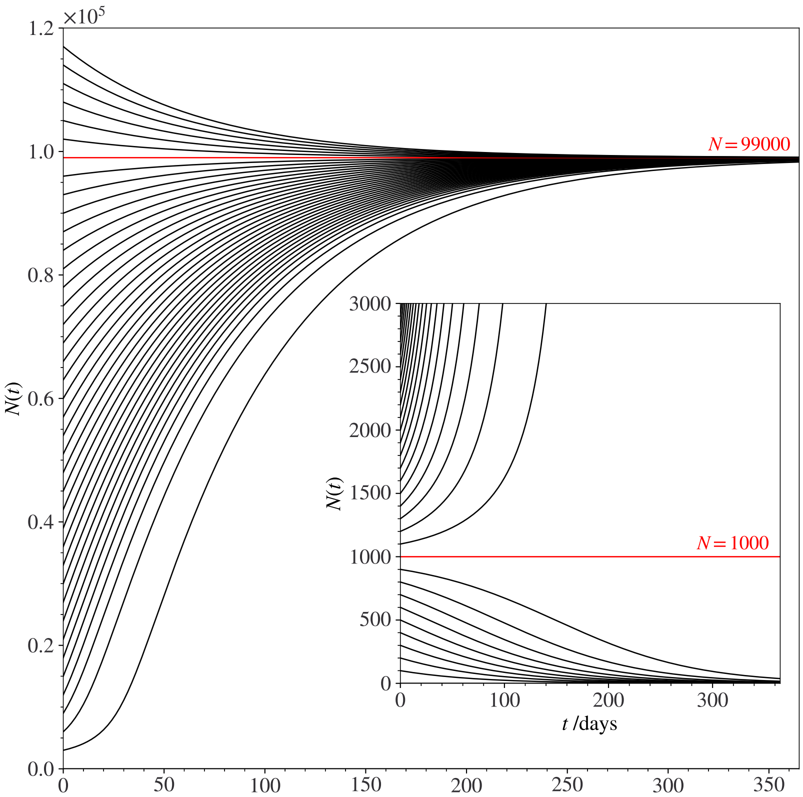


Figure 3: Population change in 1 year with different initial population with Allee effect

Figure 3 demonstrates the relationship between different initial population and its impact on population growth over time. The two red lines indicate the carrying capacity (99000) and the Allee threshold (1000 bees); these two population points are the points where the population will not grow or shrink. The critical point, or Allee threshold, is set at 1000 bees [5], below which the colony will collapse and above which it can flourish. As there were not enough adult workers in the colony to maintain the eggs' survival rate, the population below 1000 in this graph steadily shrank to zero, but the population above 1000 thrived and eventually reached a colony of 99000 bees. The carrying capacity of the population, or the most number of individuals the ecosystem can support, is 99000. Therefore, all initial population above the Allee threshold will eventually approach to the carrying capacity of 99000 bees.

2.3 With Virus

2.3.1 Model Setup

Deformed Wing Virus is a persistent pathogen that has become synonymous with the death of mite-infested colonies across the world. When DWV is injected into developing pupa, this causes a reduction in adult lifespan of 50-75%, which lead to 2 to 1.5 times the original death rate (0.02). We introduce the virus on the 400^{th} day in our model. By applying the death rate influenced by DMV to our model and changing the transition rate, we find out that the system will finally reach an equilibrium (symbiosis or extinction).

In order to determine how the colony population is affected by the virus, this model "compartmentalizes" the population, each honeybee in the population is in exactly one of the three groups:

- N_S: the population that is susceptible to the disease (namely, honeybees who have not had the infection or recovered from the infection).
- N_I: the population infected with the virus and are themselves infectious.

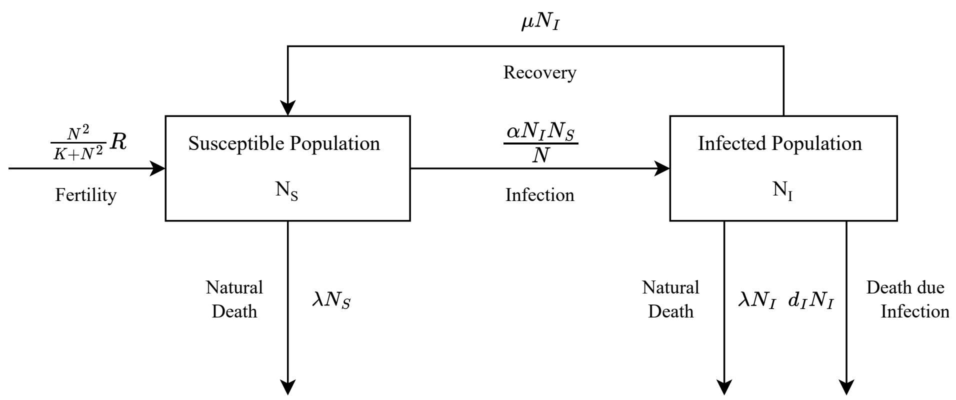


Figure 4: The flowchart of the virus model

We assume that a disease is spreading through a population of growing size N, and N_S, N_I, the number of honeybees in the two groups respectively. Individuals can move from $N_S \rightarrow N_I$(infection) and from $N_I \rightarrow N_S$(recovery), so N_I and N_S can all change with time.

The dynamics of the colony population can be specified with three parameters:

- α, the transmission rate, which is the number of contacts a honeybee has each probability of catching the disease from contact with an infected honeybee
- μ: the probability of recovery when infected
- d_I: the incremental probability of death when infected, in addition to the probability of natural death. This means the average length of the infection is $\frac{1}{\lambda+\mu+d_I}$, where death due to infection and natural causes and the recovery rate are considered.

With these parameters, the dynamics of the honeybee population with virus is summarized in Figure 4.

The population growth rate can be decomposed into:

$$\frac{dN}{dt} = \frac{dN_I}{dt} + \frac{dN_S}{dt}$$

According to the dynamics illustrated in Figure 4, the dynamics of thesusceptible population can be expressed as:

$$\frac{dN_S}{dt} = \frac{N^2}{K + N^2}R - \lambda \cdot N_S - \frac{\alpha \cdot N_I \cdot N_S}{N} + \mu \cdot N_I,$$

where μ is the probability of recovery when infected, μN_I represents the population recovered from the infection per unit time. λ is the probability of natural death, λN represents the population that died from natural causes. The fertility rate $\frac{N^2}{K+N^2}R$ and the recovery number μN_I give rise to the susceptible population, assuming all the newborns are uninfected. The natural deaths λN_S and infections $\frac{\alpha \cdot N_I \cdot N_S}{N}$ reduces the susceptible population.

The infected population may be stated as:

$$\frac{dN_I}{dt} = \frac{\alpha \cdot N_I \cdot N_S}{N} - d_I \cdot N_I - \mu \cdot N_I - \lambda \cdot N_I,$$

where α is the transmission rate, which is the number of contacts a honeybee has each day times the probability of catching the disease from contact with an infected person, we model the probability at which the susceptible adult honeybees are virus infected by the virus as $\frac{\alpha \cdot N_I}{N}$. Thus, the infections in total can be expressed as $\frac{\alpha \cdot N_I \cdot N_S}{N}$.

By combining the susceptible and infected population, $\frac{dN}{dt} = \frac{dN_I}{dt} + \frac{dN_S}{dt}$ can be elaborated as:

$$\frac{dN}{dt} = \frac{N^2}{K + N^2}R - \lambda \cdot N - d_I \cdot N_I$$

Where d_I is the probability of death caused by infection, $d_I N_I$ represents the population that died due to the virus infection. The infected population $\frac{\alpha \cdot N_I \cdot N_S}{N}$ grows when more bees are infected. The infected population drops when bees die of natural causes(λN_I), infections($d_I N_I$), or recover from the infection(μN_I).

Further analysis of the how transition rate and death rate would influence the population would be elaborated in the sensitivity analysis.

2.3.2 Parameter Calibration and Model Results

The death rate was set at 0.04 during parameter estimation, meaning that the infected bees live an average of 25 days. Since the DMV can reduce an infected bee's lifespan to 1733 days, it has been determined that a lifespan of 25 is the average. The transition rate is set at 0.4, indicating that 2.5 days are required for an infected bee to spread throughout a congested hive. We maintain the recovery rate at a constant 0.01, which is negligibly low when compared to the mortality rate.

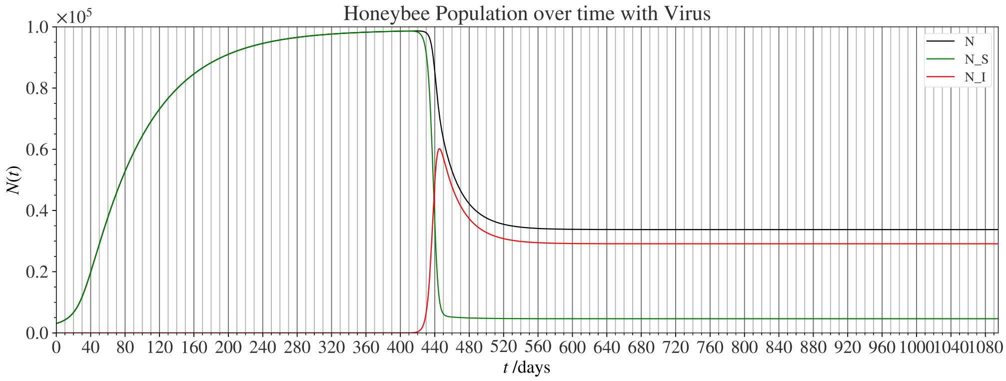


Figure 5: Honeybee Population over time with Virus

2.4 Seasonality

In the baseline model, the number of eggs laid by the queen and the lifespan of the honeybees are taken to be the average over the entire year. However, in reality both of those variables change over time.

2.4.1 The Lifespan With Seasonal Change

The lifespan of the honeybee varies with seasonal change. Most honeybees labor themselves to death throughout the summer due to the heavy workload, which reduces their longevity. Honeybees may live longer in fall and winter (for a lifespan of four to six months). The probability of individual death λ is inversely proportional to the expected lifespan m, which can be expressed by the equation:

$$\lambda = \frac{1}{m}$$

Since the lifespan of the honeybee is longer during autumn and winter, we assume the longest lifespan m_{max} occurs on 22 December, which is estimated to be the coldest temperature. Similarly, we assume that the shortest lifespan m_{min} occurs on 22June in the summertime. Thus, the function of lifespan m with respect to time t in days is a cosine function. This is given by equation 3:

$$m(t) = a_m \cos\left(\frac{t + \varphi_0}{Y} \cdot 2\pi\right) + (m_{max} - a_m) \tag{3}$$

In this case, φ_0 is the horizontal translation of the function. φ_0 is defined as the time interval between t_0 and the last 22 December, where t_0 is the starting date of population change. Y represents a year, which is 365 days. m_{max} is the longest lifespan of the honeybee. a_m is the amplitude of the function, which is:

$$a_m = \frac{m_{max} - m_{min}}{2}$$

Here, the period of the function is 365 days. m_{max} is the longest lifespan of the honeybee and m_{min} is the shortest lifespan of the honeybee, so the amplitude is expressed.

The probability of individual death λ is reciprocal of the lifespan m, which could be expressed by:

$$\lambda = \frac{1}{a_m \cos\left(\frac{t+\varphi_0}{Y} \cdot 2\pi\right) + (m_{max} - a_m)} \tag{4}$$

Substitute Equation 4 to the baseline model (Equation 1), we get:

$$\frac{dN}{dt} = \frac{N^2}{K + N^2}R - \frac{N}{a_m \cos\left(\frac{t+\varphi_0}{Y} \cdot 2\pi\right) + (m_{max} - a_m)}$$

2.4.2 The fertility with Seasonal change

The average number of eggs laid by the queen bee per day (R) varies with seasonal change. The number of eggs laid R increases in warmer seasons and decreases in colder seasons. Thus, we assume that R reaches its maximum on 22^{nd} June and reaches its minimum on 22^{nd} December. The function of the number of eggs laid R with respect to time t in days is a cosine function which is similar to what we have got for the lifespan with seasonal change. The function is expressed by:

$$R = -a_R \cos\left(\frac{t+\varphi_0}{Y} \cdot 2\pi\right) + (R_{max} - a_R)$$

In this case, φ_0 is the horizontal translation of the function. φ_0 is defined as the time interval between t_0 and the last 22^{nd} December, where t_0 is the starting date of population change. Y represents a year, which is 365 days. R_{max} is the highest fertility rate of the queen bee. a_R is the amplitude of the function, which is equal to:

$$a_R = \frac{R_{max} - R_{min}}{2}$$

There is a negative sign before the amplitude because this is the reflection across the x-axis of the lifespan function. The lifespan is the longest in winter as the temperature is the lowest and the queen rarely lays eggs, so R has become R_{min}.

Combining seasonality changes of both the lifespan and the fertility rate, we have that the final model incorporating seasonal changes can be expressed as

$$\frac{dN}{dt} = \frac{N^2}{K+N^2}\left(-a_R \cos\left(\frac{t+\varphi_0}{Y} \cdot 2\pi\right) + (R_{max} - a_R)\right) - \frac{N}{a_m \cos\left(\frac{t+\varphi_0}{Y} \cdot 2\pi\right) + (m_{max} - a_m)}$$

2.4.3 Parameter Calibration and Model Results

Our seasonality model features a critical threshold and a carrying capacity, similar to the baseline model. The population above the critical threshold thrives, whereas the population below it collapses. The second graph demonstrates how the population will gradually reach a carrying capacity (the maximum population that the environment is able to support) with different initial populations. In addition, our model's seasonality is demonstrated by the identical growth patterns in years 2 and 3.

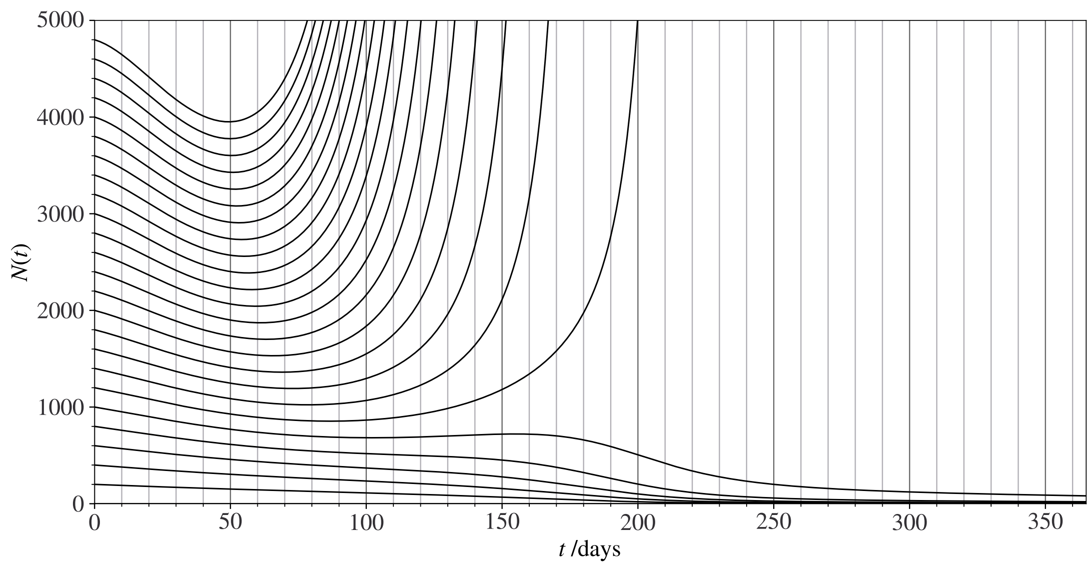


Figure 6: Population change in 1 year with different initial population starting on 1 January

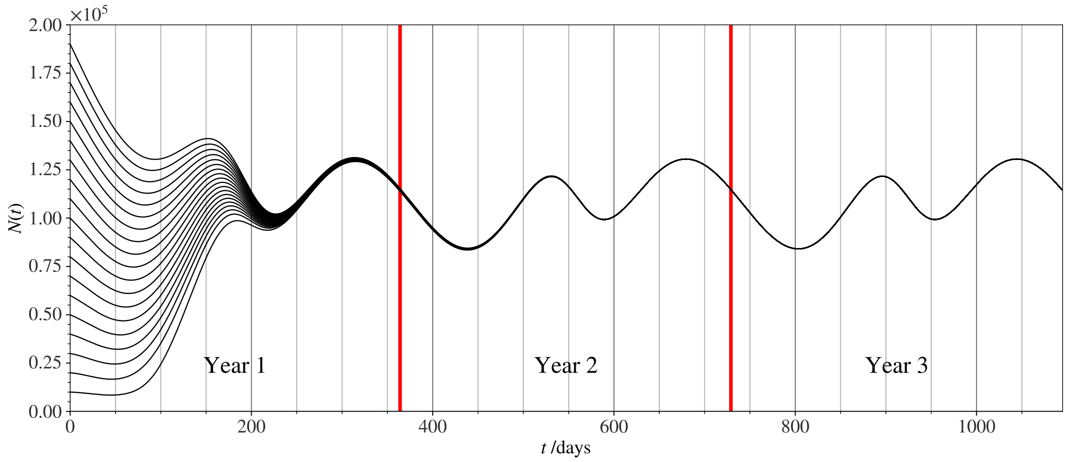


Figure 7: Population change in 3 years with different initial population starting on 1 January

We display the graphs below to show how the starting date (t_0) affected the initial population. Varied t_0 results in different initial population values. Despite the population's diverse starting positions, every graph has revealed a consistent, recognizable pattern. The curves with various starting dates are identical when translated horizontally. The curves are the same regardless of starting dates or initial populations and have a periodicity of 365 days (or one year). This illustrates the stability and validity of our seasonality model.

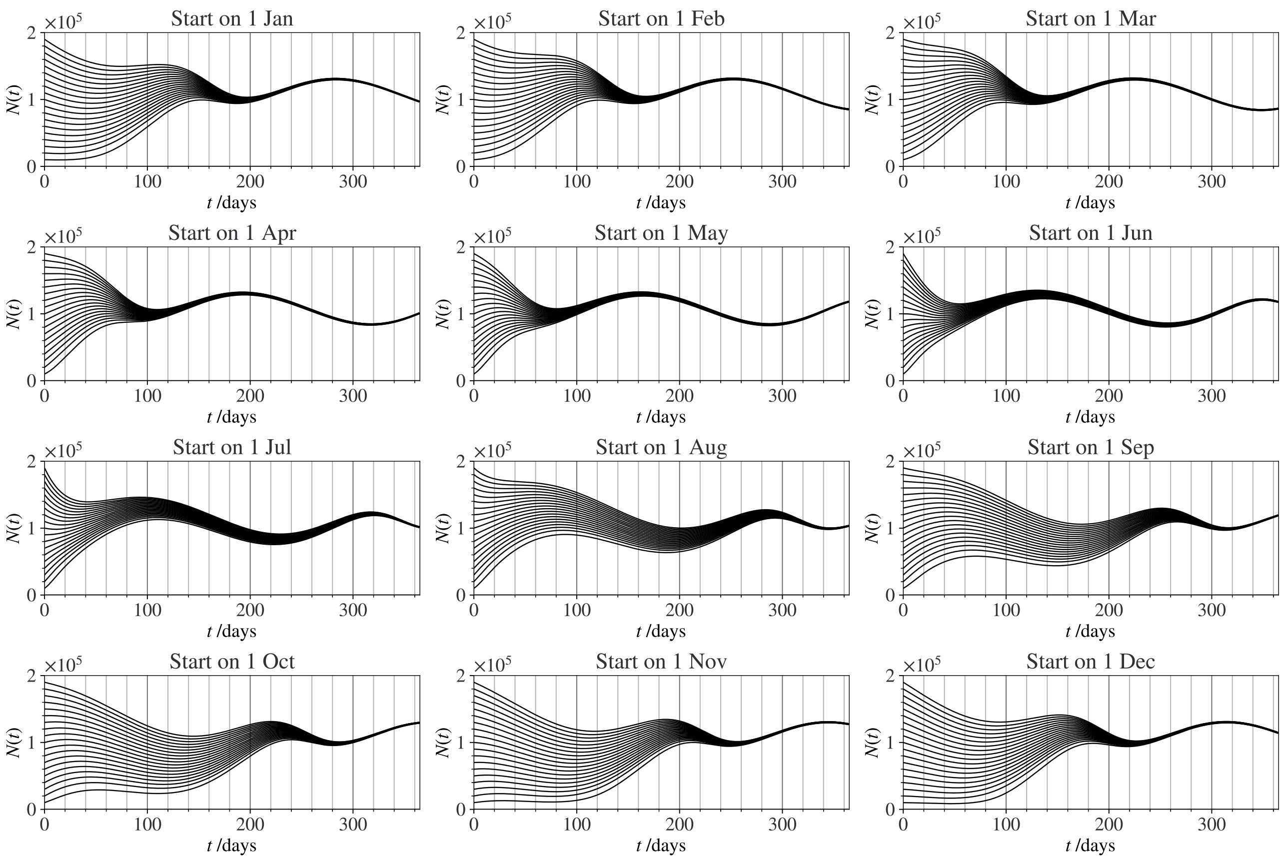


Figure 8: Population change in 1 year with different initial population starting on different dates

From the two graphs below, it can be implied that the population has a periodicity of one year: there is no difference between the three years regarding the pattern of the population change. Moreover, with

Figure 9 and Figure 10 with different initial dates of January and July, the difference is a horizontal translation, which indicates that the two graphs have the same pattern. This demonstrates the seasonality of our model.

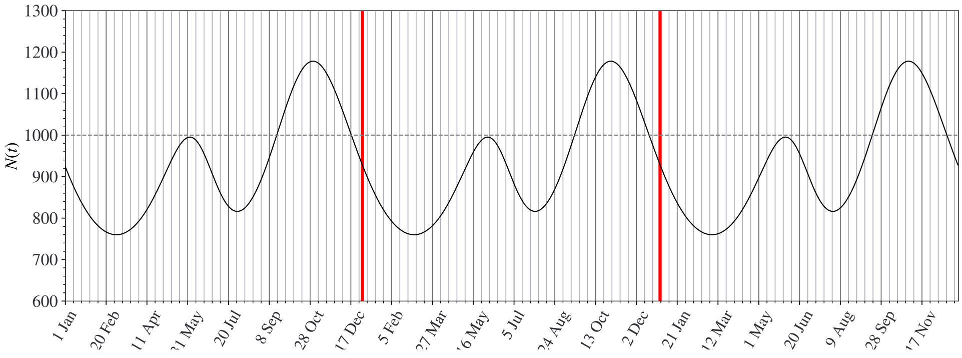


Figure 9: Critical curve starting on 1 January

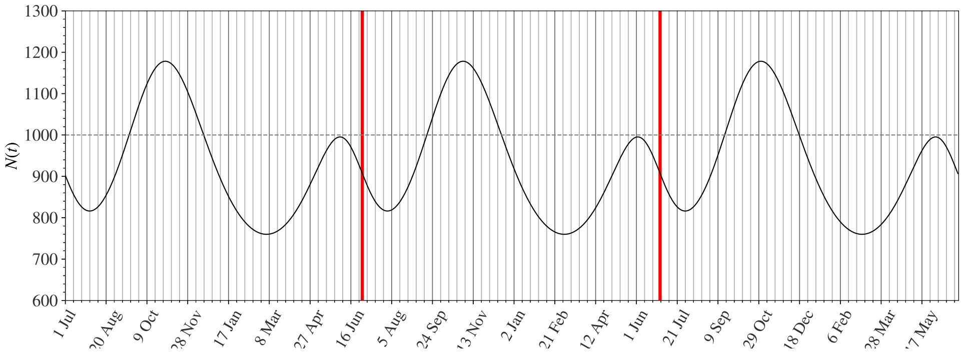


Figure 10: Critical curve starting on 1 July

This graph above illustrates the crucial point at 923 bees, below which the colony will collapse. The oscillations are larger than those of the previous initial graph because the initial population is relatively near the critical point. The curve pattern in three years is identical, implying the seasonality of our model is valid. In our seasonality model, the allee threshold (critical point) is 923 bees, while the carrying capacity is 96203 bees. These two numbers show that our model is robust and reliable because they are comparable to the baseline model's carrying capacity of 99000 bees and an allee threshold of 1000 bees.

As shown in the graph below, the seasonality function also has a maximum population. The graph's cycle is one year long, demonstrating the seasonality of our model.

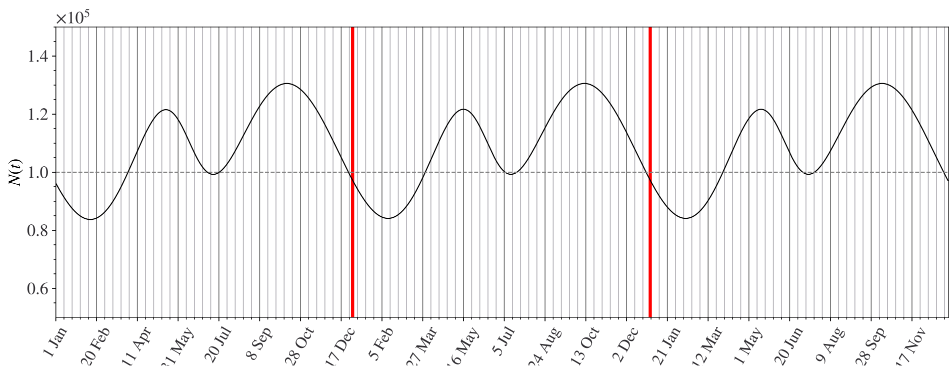


Figure 11: Maximum population curve starting on 1 January

The new average lifespan of the model with seasonality can be calculated using the method of dividing the sum of the population each day by the total birth in a one-year period when the population is relatively stable. This method can be interpreted as adding up all the lifespan of each individual, and divided by the total number of individual bees. We selected the third year from the start as the one-year time period with stable population, which can be justified by Figure 7.

In our model with seasonality, the average lifespan calculated this way is roughly 73 days, which is very close and comparable to the 67 days of average lifespan in the baseline model.

3 Sensitivity Analysis

We divide sensitivity analysis into two parts-the variables selected and the percentage change regarding the alteration of the value of the variables.

3.1 Baseline

We conducted a sensitivity analysis regarding our baseline model. We selected natural death rate λ, the number of eggs laid daily R, and K as variables. We altered the variables ±10%, and the resulting percentage changes of the model are around ±10%, meaning our variables have a linear relationship with our baseline model, proving our model's stability and credibility.

Parameter	Ratio to standard value	Critical population	Maximum population	Days to reach 90% of maximum population
standard	100%	1,000	99,000	187
λ	90%	898	110,213	202
λ	110%	1,102	89,807	174
R	90%	1,114	88,886	192
R	110%	907	109,093	183
K	90%	899	99,101	183
K	110%	1,101	98,899	191

Parameter	Ratio to standard value	Change of critical population	Change of maximum population	Change of days to reach 90% maximum population
standard	100%	0.00%	0.00%	0.00%
λ	90%	-10.17%	11.33%	8.02%
λ	110%	10.24%	-9.29%	-6.95%
R	90%	11.38%	-10.22%	2.67%
R	110%	-9.25%	10.19%	-2.14%
K	90%	-10.09%	0.10%	-2.14%
K	110%	10.11%	-0.10%	2.14%

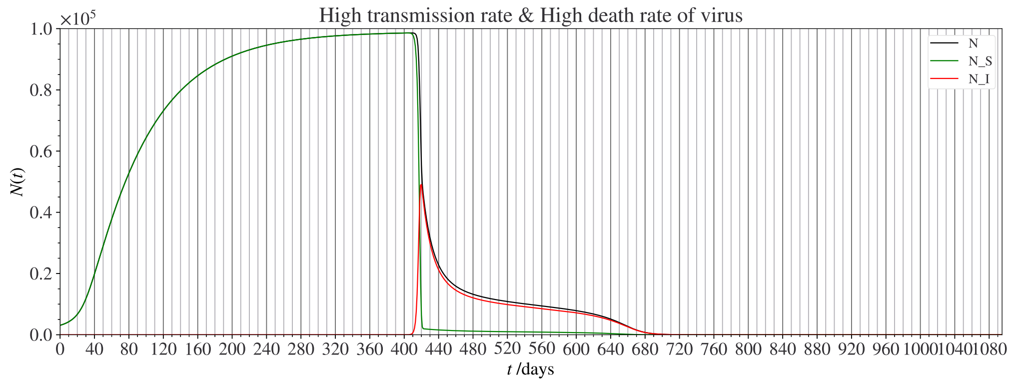
Parameter	Ratio to standard value	Population change in 1 year	Population change in 2 years	Population change in 3 years
standard	100%	3200%	3200%	3200%
λ	90%	3500%	3600%	3600%
λ	110%	2900%	2900%	2900%
R	90%	2800%	2900%	2900%
R	110%	3500%	3500%	3500%
K	90%	3200%	3200%	3200%
K	110%	3200%	3200%	3200%
Parameter	Ratio to standard value	Change of population change in 1 year	Change of population change in 2 years	Change of population change in 3 yeasr
standard	100%	0.00%	0.00%	0.00%
λ	90%	11.19%	11.67%	11.68%
λ	110%	-9.33%	-9.57%	-9.58%
R	90%	-10.60%	-10.54%	-10.54%
R	110%	10.57%	10.51%	10.51%
K	90%	0.15%	0.11%	0.11%
K	110%	-0.16%	-0.11%	-0.11%

Table 2: Baseline model sensitivity analysis

3.2 Virus

The demise of mite-infested colonies has come to be associated with the persistent infection known as Deformed Wing Virus (DMV) [13]. When DWV is injected into the pupa during development, this shortens adult life expectancy by 50%-75%, which results in 2-4 times the initial death rate (0.015). So we estimate the death rate caused by the virus would be 0.04. On the 400^{th} day in our model, the virus is introduced as the colony population has reached a relatively stable state. Applying the DMV-influenced death rate to our model and adjusting the transmission rate results in the conclusion that the system will eventually reach an equilibrium (survive or extinction).

We have created four scenarios where the death rate and transmission rate are individually high or low in order to assess the total impact of these rates on the population amount in a stable state. The conclusion suggests that, in comparison to the transmission rate, the death rate has a more significant impact on the steady-state population, which is consistent with the real-life situation.



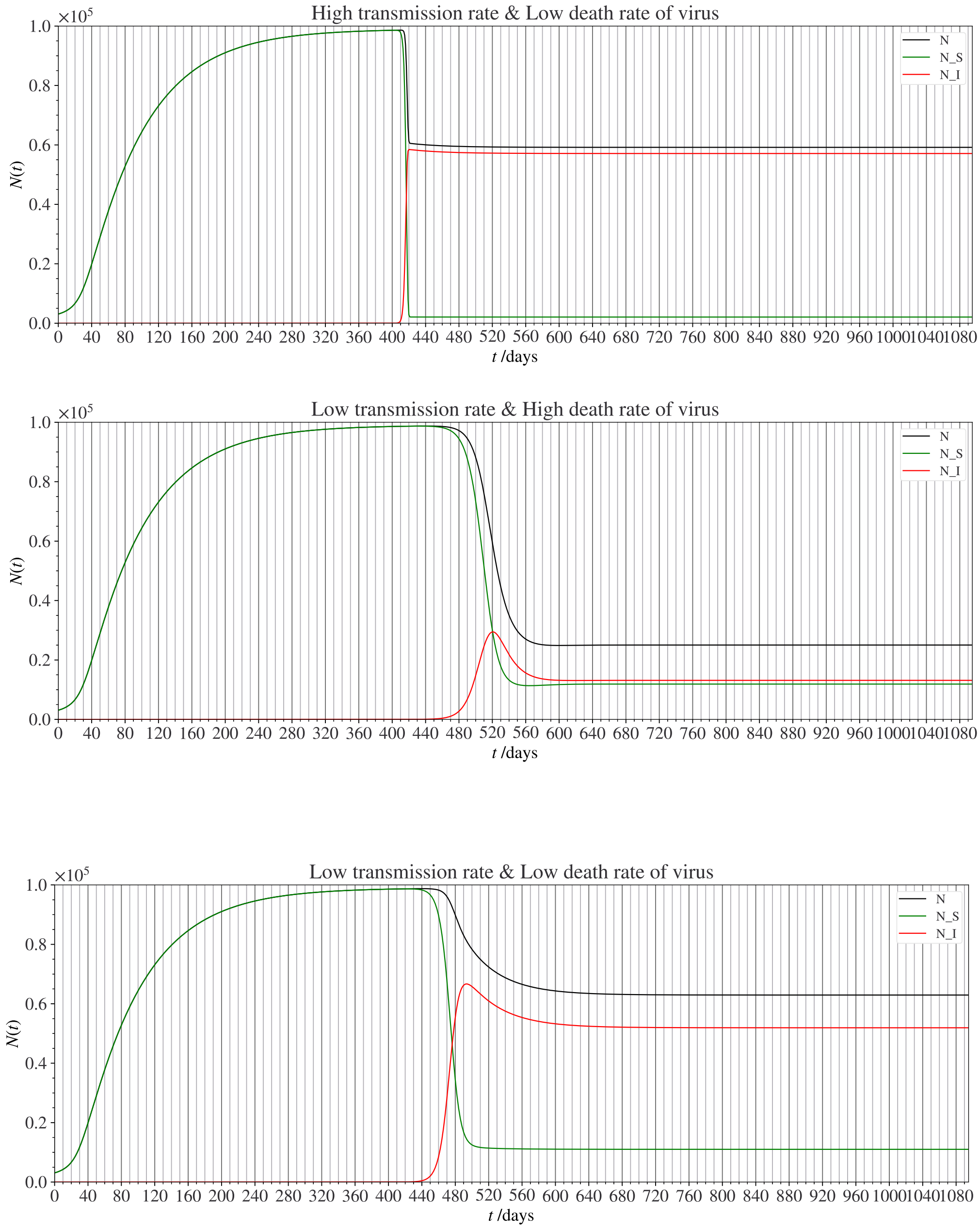


Figure 12: Comparison between different transmission rate and death rate of virus

In order to determine how the death rate or the transmission rate separately has an effect on the colony population, we made each of these rates into controlled variables. While controlling the death rate of the virus, since it shortens 50%-75% lifespan of bees, the corresponding death rate is 0.03-0.06, and we decided the death rate of the virus to be 0.04 as a medium rate in the range. Results have shown that the population will decline in death or transmission rates. The percentage change of the population is decreasing yet when the transmission rate is large to a certain value, the percentage change will become relatively constant; this situation does not occur in the death rate. With the increase in the death rate, the percentage change decreases and then increases. This phenomenon suggests that whereas an increase in the death rate affects the population continuously, an increase in the transmission rate has a diminishing effect on the population after it reaches a certain point.

Transmission rate	Death rate	Equilibrium population	Days needed	Percentage change
0.1	0.04	42,891	419	\
0.15	0.04	34,300	285	20.03%
0.2	0.04	30,893	270	9.93%
0.25	0.04	29,070	257	5.90%
0.3	0.04	27,932	249	3.91%
0.35	0.04	27,156	244	2.78%
0.4	0.04	26,588	240	2.09%
0.45	0.04	26,157	233	1.62%
0.5	0.04	25,820	235	1.29%
0.55	0.04	25,545	232	1.06%
0.6	0.04	25,320	229	0.88%
0.65	0.04	25,131	227	0.75%
0.7	0.04	24,970	223	0.64%
0.75	0.04	24,832	223	0.55%
0.8	0.04	24,711	221	0.49%

Table 3: Sensitivity analysis of transition rate
*Note that the first percentage change is considered invalid as the percentage change from 0.0 is different compared to other changes. Same applies to the next table.

Transmission rate	Death rate	Equilibrium population	Days needed	Percentage change
0.4	0.02	42,816	285	\
0.4	0.025	37,363	270	12.74%
0.4	0.03	33,051	259	11.54%
0.4	0.035	29,535	245	10.64%
0.4	0.04	26,588	240	9.98%
0.4	0.045	24,064	233	9.49%
0.4	0.05	21,850	234	9.20%
0.4	0.055	19,869	234	9.07%
0.4	0.06	18,053	237	9.14%
0.4	0.065	16,337	244	9.51%
0.4	0.07	14,633	267	10.43%
0.4	0.075	12,771	314	12.72%
0.4	0.08	4	679	99.97%
0.4	0.085	4	343	0.00%
0.4	0.09	4	275	0.00%

Table 4: Sensitivity analysis of death rate

We controlled the transmission rate to find the least death rate that can cause extinction. The findings showed that the death rate has a greater impact on colony population extinction, and the higher the transmission rate, the lower the death rate required to bring a colony to extinction. As time goes on, the death rate decreases and becomes slower. The time to reach an equilibrium state of the population has small oscillations, but overall it decreases as the transmission and death rate rise. The death rates we gained are around 0.06, which is similar to the death rate of DMV in real life, which proves our model to be effective.

Transmission rate	Threshold death rate of extinction
0.5	0.074
0.55	0.073
0.6	0.072
0.65	0.071
0.7	0.07
0.75	0.069
0.8	0.0684
0.85	0.068
0.9	0.0677
0.95	0.0675
1	0.0673

Table 5: Threshold death rates of extinction under different transmission rates

3.3 Seasonality

To assess the sensitivity of the longest lifespan of the honeybee and the maximum fertility rate of the queen bee, we calculated the percentage change of the model after adjusting the values.

We selected m_{max} and R_{max} as variables. We did not select a_m as the amplitude of the model is relatively the same when it is decreasing or increasing, and considering the change in a_m will result in the change of lifespan, making the fertility rate negative, we did not choose a_m and the percentage change of m_{max} or R_{max} to shrink 10%.

According to the sensitivity analysis shown below, the oscillation of the model brought on by altering K is adequate when K changes by 10% and -10%, the change of maximum population and critical population changes around 0.1% and 10%, respectively, which suggests that our model is stable. A similar linear relationship between the model's output and the percentage change of the variables was produced when m_{max} and R_{max} were changed. Although adjusting m_{max} caused the model's outcome to vary by a large factor, this is due to the initial dates being different, thus the percentage change is actually about smaller of the model's result.

Parameter	Ratio to standard value	Average change of maximum population with different starting dates	Average change of critical population with different starting dates
standard	100%	0.00%	0.00%
m_{max}	110%	31.39%	-23.56%
	120%	61.67%	-37.63%
R_{max}	110%	24.58%	-19.03%
	120%	49.11%	-31.43%
K	90%	0.13%	-10.07%
	110%	-0.12%	10.09%

Table 6: Impact of transmission rate and death rate to extinction

To assess how well the seasonality functions, we compared it to the baseline model. As the curve is extremely close to the equilibrium curve in year 3, indicating that it has nearly attained the equilibrium state, we determined the average lifespan in year 3. The average lifespan of bees can be calculated by dividing the average daily population by the average birth rate since one population corresponds to a new day of lifespan. The average lifespan, $\frac{1}{\lambda}$, yields a value of 0.0137, which is close to the baseline model's value of 0.015. This demonstrates the accuracy and reliability of our model.

3.4 Discussion

Overall, the longest life span m_{max} (the smallest λ) has the greatest impact on our model, while R_{max} also has a substantial influence on our model. This is owing to the extended lifespan and rising fertility will allow the population to accumulate, having a dramatic effect and causing a disproportionately large response in the overall population growth.

4 Pollination Prediction

4.1 Variables

Variable Symbol	Meaning
$H_{resource}$	The amount of honey that an area is able for bees to collect
C_{nectar}	The amount of nectar that one flower secretes in one day
C_{flower}	The density of flower in unit area
A	The area of the orchard
h	Nectar to honey production rate
p	The percentage of the amount of honeybee-produced-honey with all the honey produced
ρ_{flower}	The number of flowers on one tree
a	The number of trees in unit area
H_{hive}	The total amount of honey a hive can collect
F	Population of foragers
t_{flight}	The average time of each foraging flight
k	The proportionality coefficient of foraging efficiency
N	Population size of a hive
n_{hive}	The number of hives needed by a certain area of crops
T	Temperature in $^\circ F$
η	The probability of foraging bees that go out
δ	Ratio between the foraging time that has air pollution and the standard foraging time
a_c	Transition rate
N_U	Uncontaminated bees
N_C	Contaminated bees
λ_c	Extra death rate caused by pesticide

Table 7: Variables in the pollination prediction model

4.2 Model

To predict the amount of honeybee hives are required to support the pollination of a 20-acre parcel of land containing crops that benefit from pollination, we assumed the amount of honey is proportional to the number of pollination, we chose the amount of honey production to quantify the process of pollination. The mass of a hive's honey can be evaluated more with ease compared to using pollination.

The amount of honey that an area is able to produce can be represented can be described as:

$$H_{resource} = C_{nectar} \cdot C_{flower} \cdot A \cdot h \cdot p$$

Where the amount of honey that an area is able to collect can be represented as $H_{resource}$, with the area of the orchard presented as A, C_{nectar} as the amount of nectar one flower has, and C_{flower} as the density of flower in unit area, h as the production rate from nectar to honey, and p as the percentage of nectar collected by honeybees with all the nectar secreted.

The amount of flowers in a unit area can be expressed as:

$$C_{flower} = \rho_{flower} \cdot a$$

With ρ_{flower} as the number of flowers on one tree and a as the number of trees planted in a unit area.

The total amount of honey a hive can collect can be described as H_{hive}. We assume it is proportional with the number of foragers and inversely proportional to the time consumed for one foraging flight. The amount of honey can be expressed as:

$$H_{hive} \propto F \cdot \frac{1}{t_{flight}}$$

Where F is the population of foragers, t_{flight} is the time of each foraging flight (in days). Then, k can be set as a proportionality coefficient and the relationship between F and H_{hive} can be expressed as

$$H_{hive} = kF \cdot \frac{1}{t_{flight}}$$

Moreover, the population of foragers F can be expressed as $F = f \cdot N$, by assuming the number of foragers is proportional to the total population of honeybees in a hive. f is the fraction of foragers in the total population and N is the total population.

So, when the number of hives needed by a certain area of crops, n_{hive}, can be stated as:

$$n_{hive} = \frac{H_{resource}}{H_{hive}}$$

which the elaborated function of n_{hive} can be:

$$n_{hive} = \frac{C_{nectar}\rho_{flower}aAhp}{kfN\frac{1}{t_{flight}}}$$

4.3 Values of parameters

Each bee forage for food with 10 flights per day [17], so the average time of each flight is 0.1 days, h=0.4,p=0.35 [12], and f is 20% [24]. The bee population N of a typical hive we selected is 60,000 since most of the blossoms occur in spring.

We calculate the value of k by implementing the $H_{resource}$ and number of hives which is 51, into our model [12]. The value of k we obtained is 4.17×10^{-5}. Then, we utilize these parameters to calculate the number of hives needed for various types of crops in 81000 square meters.

4.4 Number of hives for different crops

The first crop we selected is apple trees. For apple orchards, the amount of nectar produced by each apple flower per day is 2×10^{-6} kg [19]. The number of flowers on one apple tree is 410 [28]. For dwarf apple trees, normally, 485 apple trees are planted per acre [2]. The number of hives required by $81000m^2$ of apple trees is 0.22 consequently, which is approximated as 1 hive.

We also select pear and watermelon as crops. For pear orchards, C_{nectar} is 8×10^{-6} kg [6], ρ_{flower} is 500 [20][1], a is 2000 per hectare [16]. For watermelons, $C_{nectar}$is 1.4×10^{-5} kg [22], $\rho_{flower}$is 250 [29], a is 4000 per acre [25]. Noteworthily, the population of a hive in watermelon orchards is selected as 40,000 since the flowering season of watermelon is in summer, when the population is low. Thus, the number of hives required for an area of $81000m^2$ in each case is 1.43, approximated to 2 hives for pears, and 7.85, approximated to 8 hives for watermelons.

For the three crops listed above, with the same area of cropland, the number of hives needed by watermelons is higher than that of pears, and both higher than that of apples, mainly due to the different spacing of plants and the amount of nectar secretion.

Since the bees can normally travel up to $6km$, the area of the 20-acre parcel of land can be all covered up by the range of bees' activity, which has a radius of 6 kilometers. Thus, the land only requires a single point of hives without arranging hives in a 2 dimensional plane, in order to meet the pollination requirement. The results we obtained can be justified by other results that 51 hives are needed by an area of radius $3km$ [12], and placement of 2-3 hives/ha is recommended as a high rate to be adequate for pollinating apples [18].

4.5 Factors influencing foraging ability

4.5.1 Temperature

Temperature	Foraging
$65°F$	100%
$63°F$	62%
$54°F$	21%
$51°F$	6%

Table 8: The relative percentage of honey bees foraging at different temperatures [24]

The foraging bee's activities would be affected by the temperature shift, slowing pollination down as a result. The percentage of foraging bees who would actually go out and forage decreases as a result of the drop in temperature [24].

The temperature has an influence on that the percentage of foraging bees that go out. The formula for how much honey one hive can collect and how many hives are required can be rewritten as follows:

$$H_{hive} = kF \cdot \frac{1}{t_{flight}} \cdot \eta_{(T)}$$

Where T is the temperature in $°F$, and η represents the probability of foraging bees that go out.

Using the power function to fit the data [24]:

$$\eta(T) = \frac{1.16 \times 10^{-17} T^{10.43}}{100} (T \leq 65°F)$$
$$\eta(T) = 1 (T > 65°F)$$

However, the η is a probability, meaning when it exceeds 100%, it should only be counted as 100%. When T is larger than 65 $°F$ the $\eta(T)$ equals to 100 percent. The derivative of n_{hive} with respect to temperature represents how n_{hive} changes when the temperature is changed.

$$\frac{dn_{hive}}{dT} = \frac{C_{nectar}\rho_{flower}aAhp}{kfN\frac{1}{\delta t_{flight}}} \cdot 8.62 \cdot 10^{16} \cdot (-10.43) \cdot T^{-11.43} = -\text{constant} \cdot \frac{1}{T^{11.43}} (T \leq 65°F)$$

By differentiating the original equation, we observe that $\frac{dn_{hive}}{dT}$ is negative, thus when temperature increases, the number of hives required decreases and vice versa. $\frac{dn_{hive}}{dT}$ is inversely proportional to T to the power of -11.43.

4.5.2 Air pollution

We find the PM mass concentration as a vital factor that would influence "honeybee vision", and reduce its efficiency of pollinating [8]. Foraging duration would increase as PM mass concentration and have an exponential relationship. The formula of the number of hives can be rewritten as:

$$n_{hive} = \frac{C_{nectar}\rho_{flower}aAhp}{kfN\frac{1}{\delta t_{flight}}\eta(T)}$$

Here, δ is the ratio between the foraging time that has air pollution and the standard foraging time.Using the data provided in the article, we can calculate δ as follows:

$$\delta = e^{0.004 \cdot C_{PM}}$$

The derivative of n_{hive} with respect to temperature represents how n_{hive} changes when the PM mass concentration is changed.

$$\frac{dn_{hive}}{dC_{PM}} = \frac{C_{nectar}\rho_{flower}aAhp}{kfN\frac{1}{t_{flight}} \cdot \eta(T)} \cdot 0.004 \cdot e^{0.004C_{PM}} = \text{constant} \cdot e^{0.004C_{PM}}$$

$\frac{dn_{hive}}{dC_{PM}}$ is positive, thus when the PM mass concentration increases, we would require more hives to complete the pollination, and vice versa.

4.5.3 Use of pesticide

When a pesticide is introduced to the habitat of bees, we separate bees into 2 categories: uncontaminated and contaminated. The uncontaminated bees are not exposed to pesticides yet the contaminated bees have.

$$\frac{dN_U}{dt} = R \cdot \frac{N^2}{K + N^2} - \lambda \cdot N_U - \alpha_C \cdot N_U$$

$$\frac{dN_C}{dt} = (1 - \lambda_C) \cdot \alpha_C \cdot N_U - \lambda \cdot N_C$$

$$\frac{dN}{dt} = R \cdot \frac{N^2}{K + N^2} - \lambda \cdot N - \lambda_C \cdot \alpha_C \cdot N_U$$

Here, a_c is the transition rate from uncontaminated bees N_U to contaminated bees N_C. It is associated with the concentration of the pesticide in the habitat, but in order to simplify the model, we set it as a constant. λ_C is the extra death rate caused by pesticides. Adding the equation between time and uncontaminated and contaminated bees, we can obtain an equation that indicates the relationship of the total population that may be exposed to pesticides. The introduction of pesticides actually decreases the total population and indirectly influences the number of hives required. We find the derivative of n_{hive} with respect to N represents how n_{hive} changes when the population is changed.

$$\frac{dn_{hive}}{dN} = \frac{C_{nectar}\rho_{flower}aAhp}{kf\frac{1}{\delta t_{flight}}\eta(T)} \cdot -N^{-2} = -constant \cdot \frac{1}{N^2}$$

$\frac{dn_{hive}}{dN}$ is negative, thus the number of hives decreases with the growth in population, and the pesticides would inhibit the speed of population growth.

5 Conclusion

In conclusion, our honeybee colony population model has some strengths that increase its practicability and weaknesses that limit its accuracy.

5.1 Strengths

- Multiple factors are considered. The Allee effect is integrated to investigate how the colony population affects its growth, the virus infection is implemented to discover how the virus impacts the population, and seasonality is examined to perceive how climate affects the bees lifespan and hence the population in general.
- Models are considered comprehensively. Carrying capacity and the Allee threshold are considered in the model to demonstrate the two critical points in population growth. Two groups of infected and susceptible populations are considered in the virus model. Moreover, the seasonality model covers the overall pattern of population growth in a year.
- Robust stability and validity. The sensitivity analysis and graph plots have shown the three models have aligned values regarding the same factor (e.g: critical point and the Allee threshold). The change in variables has an appropriate response in the model.
- Model results match real-world data to a large extent where the data are derived from academic research and real data

5.2 Limitations

- More potential factors can be considered, such as parasites and inadequate nutrition. We did not choose to investigate as there is a limited amount of time and we decided to prioritize the factors which have more influence on the colony population.
- Some pesticidesre neglected for simplifying our model. It was neglected and if it is elaborated in our model it will raise our models accuracy. betweenitem Factors are introduced to the baseline model separately and thus our model cannot analyze the iterations between different factors such as virus and seasonality.

TheNeedforBees.com

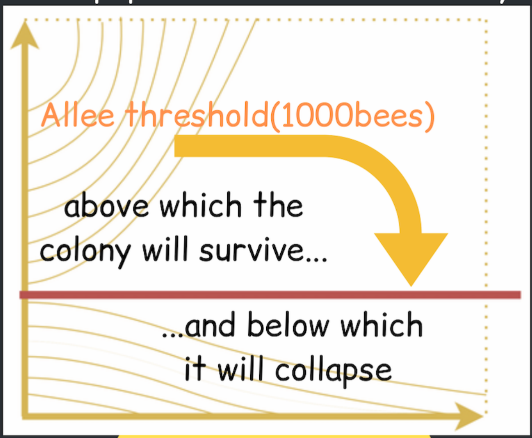
(and not just for honey!)

Honeybees have an irreplaceable role in agricultural production and the maintenance of natural ecosystems. However, honeybee populations are diminishing worldwide. There is a pressing need for understanding the key factors and crucial processes for raising colony survival rates. We present TheNeedforBees.com to improve the dire straits of honeybee survival.

Factors affecting the colony population

THE ALLEE EFFECT

HOW population affect the colony size



VIRUS INFECTION

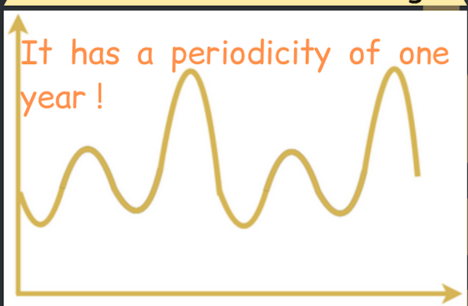
This demonstrates the honeybee population that are either susceptible or infected

the death rate has a greater impact on colony population extinction

higher the transmission rate, lower the death rate required to result in extinction

SEASONALITY

This shows how lifespan and fertility varies with seasonal change



LIFESPAN
longest on 22 Dec
shortest on 22 June
FERTILITY
→number of eggs laid
maximum on 22 June
minimum on 22 Dec

Factors affecting the population

PARAMETERS

- The natural death rate
- Lifespan
- Number of eggs laid by the queen bee
- The Allee constant
- Equilibrium population
- Virus transmission rate
- Death rate due to virus

INDICATION

Stability and validity. The sensitivity analysis has shown the three models have aligned values regarding the same factor. The change in variables has an appropriate response in the model.

CONCLUSION

The average number of eggs the queen lays per day has a substantial impact on our model as well, the longest lifespan has the greatest impact, where a 10% increase resulted in the average long-term population increase 30%. This is due to the accumulation of population caused by the two factors.

Honeybee pollination

NUMBER OF HIVES FOR DIFFERENT CROPS --SPECIFIC CASE FOR AN AREA OF 8100M^2

Apple: 1 hive
Pear: 2 hives
Watermelon: 8 hives

FACTORS AFFECTING THE NUMBER OF HIVES

Temperature:
The number of hives increases as the decrease of temperature

Air pollution:
The number of hives increases with the increase of PM mass concentration

Pesticide:
Evaluated the relationship between the contaminated and uncontaminated bees

THE MOST UPDATED HONEYBEE POPULATION MODEL IS ONLY CLICKS AWAY!

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